Observations of Atmospheric Effects on Vortex Wake Behavior

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Smoke-marked trailing vortices were generated by a light aircraft under a hierarchy of measured atmospheric stability and turbulence levels and their motion and decay was recorded photographically. Decay from both sinuous vortex interaction and core bursting type instabilities occurred, with bursting being the dominant mode. Turbulence had a strong effect on wake life, with time-to-breakup for both modes varying as $\epsilon^{1/3}$, where ϵ is the turbulent dissipation rate. Observed lifetimes ranged from 6 sec in light-to-moderate turbulence to more than 80 sec in calm, stable air. One exceptionally long-lived solitary vortex was observed for more than 3 min. Atmospheric stratification had a weak influence on wake life and its effect on wake descent could not be determined, since descent was often stopped by a rolling of the plane of the vortices. The observed data correlates well with a new theory for time-to-breakup.

Introduction

THE trailing vortices generated by a lifting surface moving through the air are left behind in a complex environment. The atmosphere is stratified vertically, since its density, temperature, and pressure vary with altitude. If this stratification is stable, then vertical motions are inhibited, while unstable stratification enhances vertical atmospheric mixing and results in turbulence. Unstable stratification is not a prerequisite for the presence of atmospheric turbulence, however, since wind shear or air flow around obstacles can result in turbulence in even the most stably stratified atmosphere. The atmospheric environment also moves laterally (wind) and vertically (thermals, updrafts, downdrafts) with various scales of motion.

Since the energy present in the atmosphere is so very great, at some point in the life of even the most energetic trailing vortex system the atmospheric factors will begin to influence and eventually dominate its motion and decay. In order to better understand some of these interactions between aircraft wakes and the atmosphere, a light aircraft (Cessna 170) was fitted with smoke generators on its wingtips and the smoke-marked vortices generated by it were photographed by cameras located directly below the flight path, directly to the side of the flight path (on a nearby mountain), and inside the aircraft.

Items of specific interest were the effects of atmospheric turbulence and stability on wake descent, vortex separation, and wake lifetime. Consequently, the atmospheric turbulence and lapse rate were measured by instrumentation on-board the airplane. In all 29 such tests were performed, with the turbulence levels encountered ranging from complete calm up to light-to-moderate turbulence and with atmospheric stabilities ranging from neutrally stable (adiabatic) to very stable (strong temperature inversions).

Description of the Experiment

The flight tests were performed near El Mirage Dry Lake in California where the Shadow Mountains to the east provided the needed elevated camera platform and a north-south dirt road acted as a tracking aid for the wake generating airplane. The vertically oriented camera installation was located on this road. A horizontal viewing camera was situated 1290m to the east on a lesser peak of the range, at an altitude of 131m above the first camera. The airplane was usually flown southbound along the road at the latter elevation and directly over the vertical cameras, although various special experiments resulted in variations of this standard pattern.

The wakes were generated by a Cessna 170, a single-engine, high-wing lightplane. The wing span of this aircraft is 11.0m and its weight (mass) in these experiments was about 910 kg, giving a span loading w/b = 811 newton/m. The computed circulation, assuming an elliptic lift distribution, ranged from Γ = 16m²/s up to 33m²/s as the forward speed ranged from a high of 56m/s to a low of 27m/s. All flights were made in level flight with flaps retracted and power set as required to maintain altitude.

The vortices thus generated were marked by colored smoke generated by U.S. Army M-18 smoke grenades which were mounted in specially modified wingtips, as shown in Fig. 1. Four smoke grenades were installed in each wing, with red grenades in the left tip and green ones in the right one. A cable from each tip to the cockpit allowed the grenade pins to be pulled sequentially allowing from 1 to 4 grenades to be used at once on each wing. The best photographic quality was obtained when two grenades per vortex were used. Figure 2 shows the residue from the smoke left behind on the underside of a wingtip and shows clearly the vortical nature of the airflow over the tip.

In addition to the smoke grenades, instrumentation was also installed on the airplane to record atmospheric parameters and the aircraft speed and altitude. The air tem-

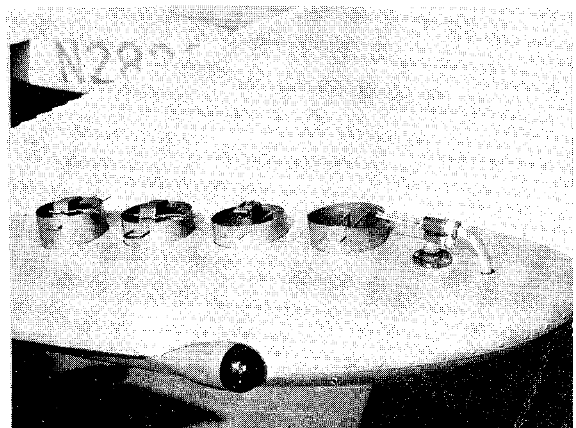

Fig. 1 Modified wingtip, showing smoke grenade installation.

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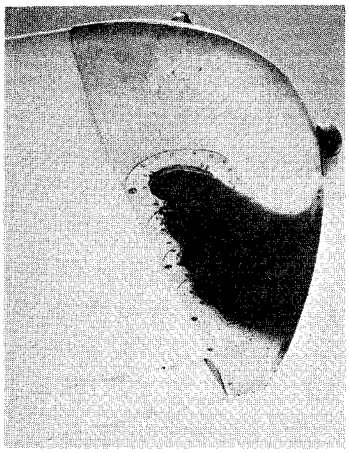

Fig. 2 Smoke residue on wingtip after flight tests.

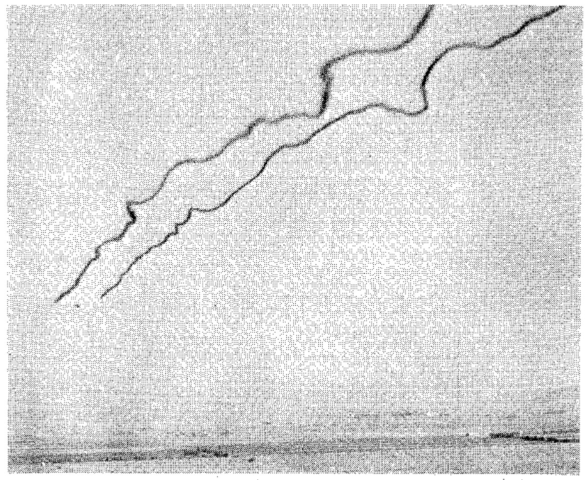

Fig. 4 Effect of turbulence on wake.

perature was measured by a thermistor vortex thermometer, pressure altitude with an electrical pressure transducer, airspeed with an electrical differential pressure transducer mounted on a specially installed pitot-static tube, and turbulence was measured with a Meteorology Research, Inc., Universal Indicated Turbulence System (described by MacCready[1]).

By flying soundings near the test area, vertical profiles of turbulence and temperature were determined. The atmospheric stability was then determined from the temperature profile. The turbulence was rated in terms of the rate of dissipation of turbulent energy into heat, ϵ, in the inertial subrange of isotropic turbulence (for an explanation of the concept, see MacCready[2]). The cube root of this quantity can be related to the "bumpiness" a pilot feels and to the accelerations experienced by him. An experimentally derived correlation between $\epsilon^{1/3}$ and the feel was obtained by Gannon, Severson, and Tombach[3] and will be used below in the presentation of the data from the experiment.

On each experiment day, the aircraft flights began as soon after sunrise as there was enough light to photograph the wake, and continued until the thermal mixing due to solar heating had eroded the predawn stable stratification and the turbulence level had increased sufficiently to severely shorten the life of the wake.

The measured wind at the flight level (determined by measurement of the speed at which the wake drifted) was generally from the west and perpendicular to the usual flight path, and its speed ranged from calm to 6.8m/s. The measured lapse rate γ at the flight level ranged from an extreme inversion (+16°C/100m) during some low level tests near the ground to the slightly unstable level of −1.3°C/100m, with the range at the normal altitudes being +1.6 to −1.3°C/100m. The measured turbulence levels varied from negligible ($\epsilon^{1/3} \sim 0.2$ cm$^{2/3}$ s^{-1}) up to light-to-moderate ($\epsilon^{1/3} \sim 2.5$ cm$^{2/3}$ s^{-1}).

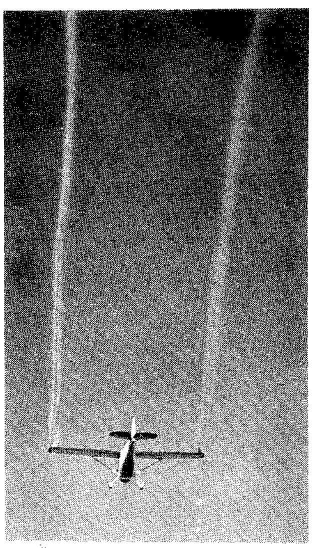

Fig. 3 Initial appearance of smoke-marked vortex trail.

Qualitative Observations of Wake

In addition to a considerable amount of quantitative data which was obtained by measurements of the photographs, and which will be reported below, there were many interesting phenomena which stood out to even a casual observer of the experiment or of a film of it. This section is devoted to a discussion of some of these "obvious" phenomena.

Figure 3 is a view of the initial appearance of the smoke trails behind the Cessna 170. The red (left) vortex appears to be smaller than the green (right) vortex, because the red smoke grenades emitted smoke only from the bottom while the green ones burned from both ends, resulting in a larger marked vortex on the green side. Some distance behind the aircraft, this "fuzzy layer" on the green vortex had diffused and both vortices were then of the same apparent size.

Atmospheric motions influenced the wake shortly after aircraft passage, often as close as three spans behind it. The magnitude of this influence varied, with noticeable effects often taking some 20–30 sec to appear in very calm air, while a few seconds would suffice in turbulent conditions. Figure 4 shows this atmospheric influence over about ½ km of wave length (foreshortened considerably by a telephoto lens). In addition to small scale wiggles of the same scale as the diameter of the smoke trail, many segments of the wake have been influenced by atmospheric eddies whose scale is larger than the vortex separation. This particular wake was photographed in a near-neutrally stratified atmosphere (γ = −1.3°C/100m) with light turbulence ($\epsilon^{1/3} \sim 2.0$ cm$^{2/3}$ s^{-1}), and it was destroyed by a sinuous instability (discussed below) about 18 sec after generation.

In a very calm and stable atmosphere the wake behaved in a very orderly manner, the two vortices descending at very nearly the same rate and with only slight waviness. In a typical case the initial descent speed was 0.62m/s (which corresponds to the theoretically predicted value); it had slowed down to 0.31m/s after 60 sec, and was destroyed 5 sec later by a burst-type instability (which will be discussed below).

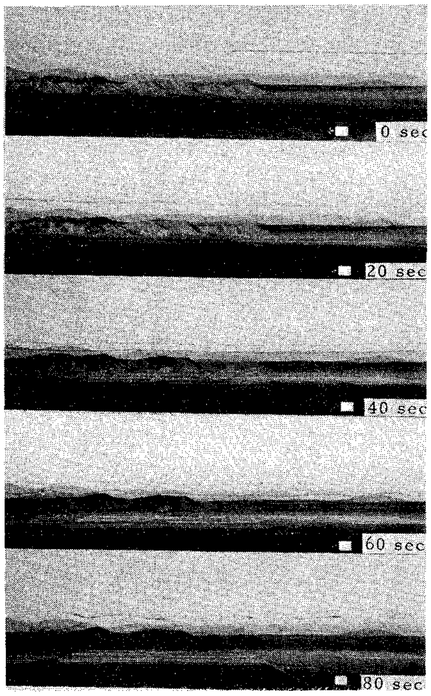

Fig. 5 Side view of wake in calm, stable atmosphere at 20-sec intervals, showing rolling tendency and destruction by core-burst instability.

The vortex system rarely behaved in this manner, however. One disturbing characteristic was the tendency of the vortex system to roll onto its side. This occurred frequently, at all levels of stability and turbulence, although the degree of roll varied. Figure 5 shows an extreme case where the wake rolled more than 90° and the average vertical vortex spacing at t = 80 sec was 1.2 times the span, or about 1.5 times the normal spacing. Burnham, et al[4] have observed similar rolling of jet transport wakes near the ground.

In both the present investigation and the study by Burnham, et al, the upwind vortex became the lower one most of the time (in 73% of the cases in the present study). There was no preference for the left or right side, hence the propeller rotation did not appear to play a role. Since, however, the Cessna 170 was usually some 130m above the ground and the measurements by Burnham, et al, were usually at lower levels, the shearing flow in the atmospheric boundary layer could influence the wake. Current investigation suggests, in fact, that shear is the cause of the roll.

The last photograph in Fig. 5 shows an often observed mode of catastrophic decay of the vortices, which was seen over the entire range of atmospheric and flight conditions encountered in this test series. This type of decay manifests itself as a localized "burst," or sudden increase in diameter, of a single vortex core, followed by rapid travel of a conical parcel of smoke down the vortex. There is usually little or no smoke left behind this traveling region, while the density of smoke within it increases as it moves down the core, suggesting that at least some of the smoke initially in the core is swept up by it. The normal burst travels along the core toward the generating aircraft, but frequently (particularly in older vortices) bursts were seen to travel away from the aircraft, or a burst would result in two of the conical regions traveling away from each other and leaving a smoke-free volume in between. Sometimes two bursts would occur on adjacent portions of the same vortex and would travel toward each other, eventually colliding and leaving behind an intensely marked disk-like parcel of smoke.

Bursting of trailing vortex cores has been observed before, particularly in small-scale water tank tests[5,6] where

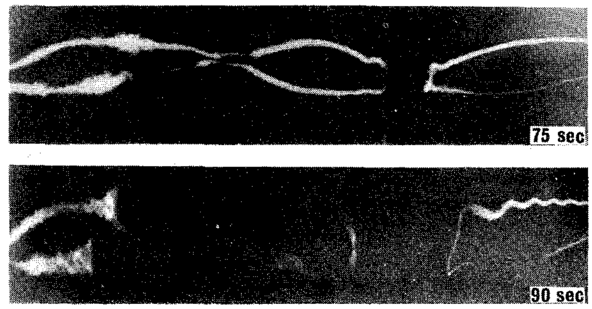

Fig. 6 Decay of B-47 contrails by bursting (left) followed by sinuous interaction (right).

it appears to be the dominant mode of vortex decay. Full-scale flight tests may also display the same form of instability,[7-9] and it has been observed in wind tunnel experiments.[10] The left side of Fig. 6 (from Ref. 8) shows details of the bursting of B-47 contrails at altitude.

The cause of the burst and the details of it are not well understood. Olsen[6] deduces that axial flow in the core plays a role. Hackett and Theisen[10] found an adverse pressure gradient in a wind tunnel diffuser to be a contributory factor. In water tank tests, both they and Widnall, et al.,[5] found that the bursting could precede or follow the sinuous type vortex interaction. Harvey and Fackrell[11] postulate that the bursting is a core-edge phenomenon related to a kinematic instability of elements of the vortex core, and show flow visualization experiments to support their tentative model.

The current experiments showed the core bursting to be independent of the sinuous instability, since bursting was observed at all points along the vortex and it did not seem to have any relation to the curvature of the vortex filaments or to their local separation. This is at variance with the observations of Widnall, et al., who state that the bursts were observed in the "high pressure region where the vortices are farthest apart" and are coupled with the sinuous instability, and of Scorer[12] and Scorer and Davenport[13] who describe bursts at the point of least separation of the vortices.

Although the core bursting instability was observed at all levels of turbulence and atmospheric stability, the sinuous (Crow[14]) vortex interaction rarely occurred at low levels of turbulence. Figure 6 showed a bottom view of vortex linking as a result of this type of instability. Figure 7 shows a greatly enlarged side view of one case observed during the current tests in light turbulence with neutral

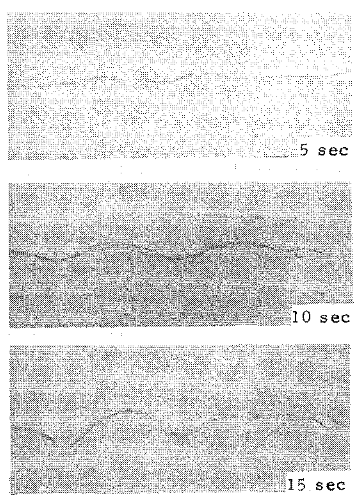

Fig. 7 Side view of growth of sinuous instability and linking of vortices.

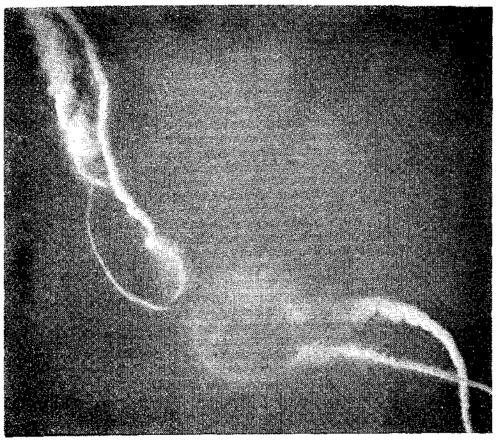

Fig. 8 Simultaneous occurrence of various forms of instability, seen from below and slightly to the side of the wake.

stability. The linking is preceded by a rapid downward acceleration of those portions of the vortex trail which are closest together, and the actual contact takes place in a near-vertical plane, resulting in saddle-shaped vortex rings.

Figure 8 is a complicated scene showing a point where the vortices have linked into loops and where the right loop has subsequently burst with the conical smoke regions moving off to the right of the picture. The left loop is well defined at the point of joining, but is convoluted and bursting at the edge of the photograph. This photograph shows the very clean nature of the linking, with no smoke residue left outside the cores of the unburst vortex at the point of contact.

Measurements of Wake Motion and Decay

Quantitative measurements of wake life were made from the motion pictures. The age of the wake at each occurrence of either a burst or linking due to a sinuous type instability was noted, with the time of the occurrence being defined by the instant at which a burst had progressed sufficiently far to leave a small clear space in the smoke trail or when the two vortices had actually made

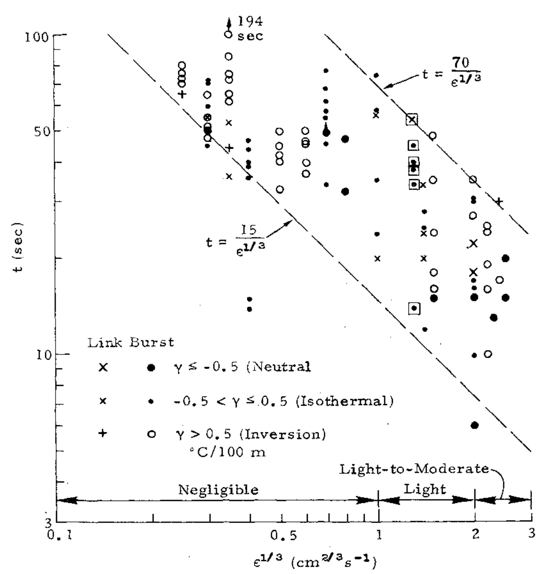

Fig. 9 Time of occurrence of observed instabilities as a function of turbulence. Γ_0 = 16m^2/s for the boxed points; otherwise $\Gamma_0 \sim$ 30m^2/s. The two arrowheads indicate that the point indicated was the last observed instability but that data is not available to give the time of final decay.

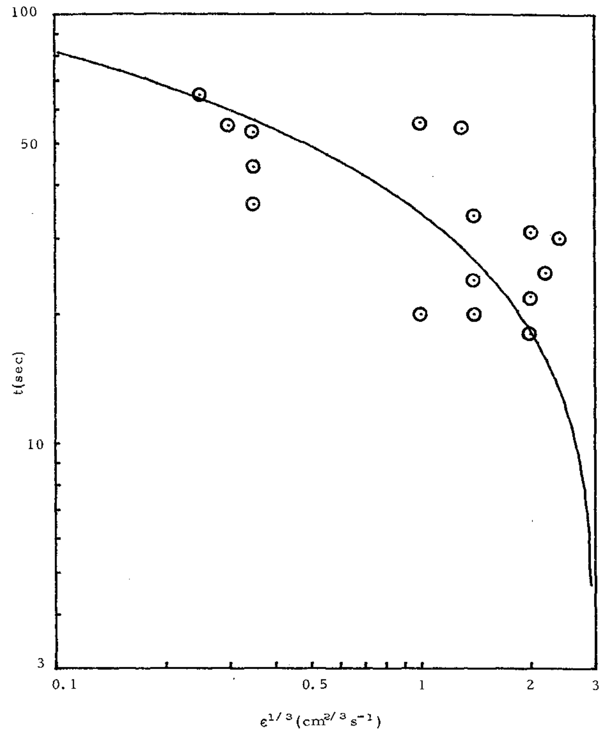

Fig. 10 Correlation between observed times of vortex linking (points) and theory of Crow and Bate (line).

contact and linked. A number of such events could take place along the length of the wake in the camera field of view and several could even occur at the same position (e.g., a burst in each vortex, or a linking followed by a burst) before the vortices were totally destroyed.

Figure 9 displays all of the observed data on all decay mechanisms up to wake destruction as a function of the atmospheric turbulence level at the altitude of the wake. Several interesting points are apparent. First, both linking and bursting points fall in the same general area, there being no general tendency for linking to precede bursting, or vice-versa, at any given turbulence level. Second, although both linking and bursting take place at all turbulence levels, linking is fairly uncommon unless there is at least light turbulence. Third, the general effect of atmospheric stratification is weak, but there is a general tendency toward increased wake lifetime with increased stability at a given turbulence level.

There is a general $1/\epsilon^{1/3}$ behavior of the data in Fig. 9. A reasonable expression defining the earliest time at which an instability was likely to occur is $t_1 = 15/\epsilon^{1/3}$ sec, which is drawn on the figure. Similarly, the time by which the wake is usually totally destroyed is represented fairly well by $t_2 = 70/\epsilon^{1/3}$ sec.

The one point marked t = 194 sec is a somewhat unusual one which carries disturbing implications for the prediction of jet transport wake lifetimes. In this case, one vortex underwent a bursting instability at an age of 65 sec. Its mate thereupon remained fixed in space (except for drift with the wind) with no apparent change in vortex structure for at least two more minutes. The camera filming this event ran out of film at t = 194 sec, so the ultimate fate of this vortex is unknown. A similar case where a B-52 vortex persisted for some 6–7 min. has been reported by MacCready,[15] and Burnham, et al.,[4] mention one boeing 474 vortex which persisted near the ground for at least three minutes. In all of these cases, the common feature was the apparent dissipation of one vortex, which then seemed to suppress the appearance of a sinuous interaction of the two vortices. Whatever mechanism inhibited the bursting of the remaining vortex is not known.

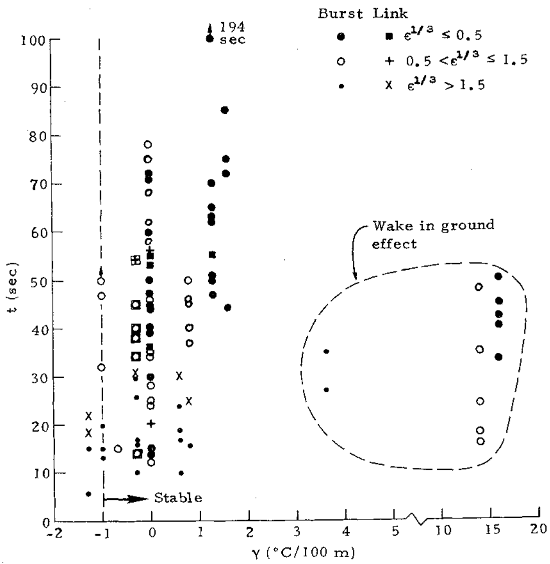

Fig. 11 Time of occurrence of wake instabilities as a function of atmospheric stratification. $\Gamma_0 = 16\text{m}^2/\text{s}$ for the boxed points; otherwise $\Gamma_0 \sim 30\ \text{m}^2/\text{s}$.

Figure 9 shows one data point set (marked with small boxes) where the initial circulation ($\Gamma_0 = 16\text{m}^2/\text{s}$) was about half that for the other points (where $\Gamma_0 \sim 30\text{m}^2/\text{s}$). Although the data is limited, it appears that there is no strong effect of circulation on the life of the wake. Crow[14] predicted that the amplification time constant for the sinuous instability should vary as Γ^{-1}, where the computed value from his formulas is 19 sec for $\Gamma \sim 30\text{m}^2/\text{s}$ and 35 sec for $\Gamma = 16\text{m}^2/\text{s}$. The data shows several vortex linkings which have taken place at wake ages of from one to two of these time constants, which is much faster than his observation that linking should require about three time constants. There is a correlation between time-to-linking and turbulence level, as has already been noted, and at low turbulence levels the observed points do approach an age of about three amplification time constants. This suggests that the purely kinematical instability treatment by Crow gives a lower bound to the rate of growth of the sinuous instability, and that turbulence provides a forcing function for the instability.

Motivated by these experimental observations, Crow and Bate have further refined Crow's original theory to include turbulence stimulated instability growth. This work, which is too lengthy to discuss here and is presented in detail elsewhere,[16] predicts the behavior shown in Fig. 10 for the conditions of these flight tests, and involves no fitted constants. As can be seen, the agreement with the experiments is quite satisfactory.

Figure 11 shows the same data as Fig. 9, but plotted against the lapse rate. The strong effect of atmospheric stability on increasing the life of the wake is quite apparent, but it must be noted that there is often a correlation between atmospheric instability and turbulence, since an unstably stratified atmosphere is generally turbulent. However, turbulence created by shearing flow can be present even under stable stratification, with the effect of the stability being to suppress the vertical spreading of the turbulent region. The degree of the dependence of turbulence on stability shows in the figure, since the dark points corresponding to low turbulence tend to appear at greater stabilities than the lighter points for greater turbulence. The cluster of points in the right half of the graph corresponds to wakes which were generated in ground effect, or which descended into ground effect, and their independent behavior from the remaining data suggests that atmospheric stability is not a major factor controlling wake lifetime. The few $\Gamma_0 = 16\text{m}^2/\text{s}$ points appear to fall in with all the others for $\Gamma_0 \sim 30\text{m}^2/\text{s}$, hence there appears to be no stability and circulation interaction governing wake life.

The descent of the wake was also measured. Several measurement points were chosen along the approximately ½ km wake length visible in the photographs and the measurements from these points were averaged. An example of the trajectories thus obtained is plotted in Fig. 12 for one test day with fairly strong stability and negligible turbulence. There is a fair amount of variability in the distance the wake descended, which is mainly due to the amount the wake rolled while descending. Shown on Fig. 12 is the largest average vertical spacing between the vortices, Δh (with negative values denoting the left vortex is below the right one), and it can be seen that passes 1, 3, and 4 with the smallest Δh had the greatest descent. For reference, the initial horizontal spacing of the vortex pair is 8.7m, so that the vertical spacing in pass 5 was some 1.5 times the original horizontal spacing.

Similar behavior was noted in other atmospheric conditions. There was a noticeable decrease in wake descent when the turbulence was greater than that for Fig. 12. On the other hand there was no discernible effect of atmospheric stability, but it may have been masked by the irregular behavior arising from the rolling tendency.

Some attempts were made to measure the vortex spacing, but, since the wake had a tendency to roll, these measurements were difficult to make. For essentially level wakes the vortex spacing was initially very nearly 0.8 times the wingspan and it tended not to vary too greatly from that value as the wake descended. The usable data is fairly meager, but if any change in vortex spacing with descent could be detected, it appeared to be a slight increase up to at most 1.2 b_0 (or about one span). As was noted earlier, though, vortex separations in excess of a wingspan were observed in wakes which had rolled completely onto their sides.

Summary of Status of Theory vs Experiment

The state of development of theoretical formulations which can describe the observed behavior varies widely.

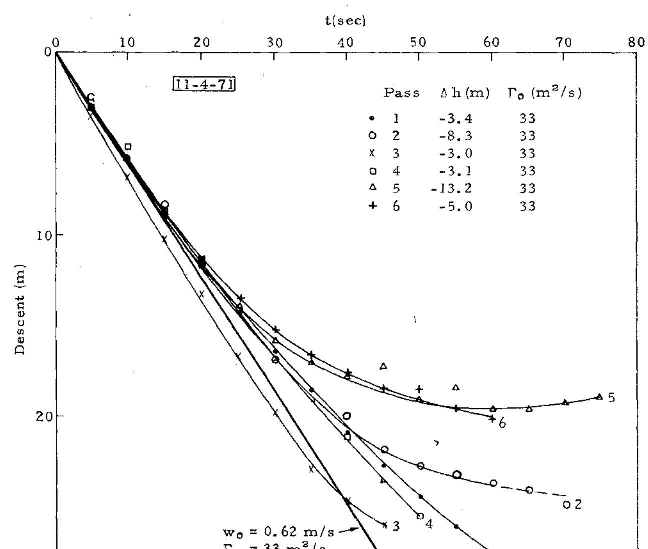

Fig. 12 Trajectories of wake descent out of ground effect in a stably stratified atmosphere ($\gamma \sim 1°\text{C}/100\text{m}$, an inversion) with negligible turbulence ($\epsilon^{1/3} \sim 0.3\ \text{cm}^{2/3}\ \text{s}^{-1}$). Δh is the maximum observed vertical vortex separation. The theoretical initial descent rate is shown. Broken lines indicate less reliable data.

Considering the instabilities first, the work by Crow[14] which was mentioned earlier describes the sinuous instability fairly well, with the amplification effect of turbulence analyzed in Ref. 16. On the other hand, there seems to be no adequate explanation of the core-burst type of decay, and observations of bursts (if they are all of the same phenomenon) appear to be contradictory. Some observations, e.g., those by Chevalier,[9] indicate that there may be two distinct types of bursts.

The over-all motion of the wake cannot be wholly explained by current theories. The rolling tendency has not yet been analytically modeled, although wind shear appears to be the causing mechanism. The author knows of at least seven efforts which have been made to consider the effect of stable stratification on wake descent.[13,16-21] The solutions fall into two categories: 1) the wake slows down and spreads out, or 2) its descent speeds up and the vortices converge. Two analyses allow both possibilities, the choice depending on atmospheric stability. The nature of the solution depends, in general, on the details of the assumed entrainment/detrainment mechanism for the buoyancy-generated vorticity, for which experiments give few clues. The present work favors the slowing-down and spreading-out theories, but the results are far from conclusive.

Conclusions

The experimental work presented has illuminated a variety of aspects of wake behavior. The following summarizes the major observations of the field program and includes some which were not presented here but appear in the comprehensive report on the study:[22]

1) The vortices were never observed to decay away due to viscous or turbulent dissipation, but were always destroyed by some form of instability.

2) Two modes of instability were observed. One was a localized "bursting" of the smoke-marked core of an individual vortex, usually with no apparent effect on the adjacent vortex or on distant segments of the same vortex. The other instability was the well-known sinuous instability of both vortices which results in their linking into vortex rings. The linking was observed to be a very "clean" process, with no visible smoke residue left behind. In these experiments, the bursting was the dominant mode of decay.

3) There is a clear correlation between wake lifetime and atmospheric turbulence. The life of the wake is dramatically shortened by even small amounts of turbulence. For these particular tests the lifetime at any particular level of turbulence tended to be the same, regardless of whether decay was due to bursting or linking.

4) As has been observed before, if one vortex is destroyed by bursting, the other vortex will sometimes last a very long time. One such case was observed where the core of one vortex was still clearly defined by smoke at an age of 194 sec.

5) There is also an apparent correlation of wake lifetime with the lapse rate but the effect is weak (if the turbulence level is fixed).

6) The majority of the wakes were observed to roll to some degree. A few rolled so much that the plane of the vortices was vertical or past the vertical.

7) Wakes out of ground effect usually descended between 1 and 2½ spans before they were destroyed by an instability. In all cases, the descent speed slowed as the wake descended, and a few wakes were observed to level off or even rise upward. There was no observable correlation between wake descent and atmospheric stability because the motion appeared to be dominated by the variable rolling tendency of the wake.

8) The vortex spacing remained essentially unchanged during the descent of the wake, with a small increase in spacing being conceivable. Rolled wakes often had larger vortex separations.

The question of scale is of relevance to the experiment series with the Cessna 170. The relationship between energy in the wake and that in the atmosphere should have some effect on determining the degree of atmospheric dominance of the wake motion. Wake instabilities are surely scale-dependent. In fact, it appears that the common mode of decay in very small scale (or low Reynolds Number) experiments is the core bursting mode, while for experiments with large transport aircraft it appears to be the linking mode. The separation is not quite clear cut, though, and both modes have been observed at all scales. The coexistence of both modes in this experimental series at essentially the same wake ages, is of interest, as is the fact the incidence of both forms of instability is coupled to the turbulence level. Applying this information to the wakes generated by large aircraft is not straightforward, however.

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On the Inviscid Rolled-Up Structure of Lift-Generated Vortices

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A simple form is presented of the relationships derived by Betz for the inviscid, fully developed structure of lift-generated vortices behind aircraft. An extension is then made to arbitrary span-load distributions by inferring guidelines for the selection of rollup centers for the vortex sheet. These techniques are easier to use and yield more realistic estimates of the rolled-up structure of vortices than the original form of Betz' theory when the span loading differs appreciably from elliptic loading.

Introduction

LIFT-GENERATED vortices behind aircraft usually consist of two adjacent, well organized, oppositely rotating flow fields which are potentially dangerous to following aircraft. It is therefore highly desirable to be able to predict the structure of such a vortex pair for a wide variety of lift configurations in order to better assess the potential hazard and to explore ways to alleviate it. A theory by A. Betz[1] uses the three conservation equations for vortex systems to relate the structure of the vortex sheet behind an isolated wingtip (isolated half span) to the structure of a single, fully-developed vortex. Although this theory does not appear to have been used extensively in the past, it has recently been demonstrated by Donaldson[2] to be useful and often more accurate than more complex methods. The favorable publicity given to Betz' method by Donaldson led to an elaboration of the theory and more examples by Mason and Marchman[3] and to the use by Brown[4] to predict the axial flow velocity in the vortex. These papers used the rollup equations in about the same form presented by Betz.

This paper presents a new form of the rollup relationships that is simpler in form and easier to use. These equations are then applied to several span-load distributions to illustrate the variations in vortex structure produced by various span-load distributions. After the method is generalized to include situations wherein the vortex sheet rolls up into several vortices on each side of the fuselage, a span-load distribution typical of current large aircraft is analyzed.

Derivation of Simplified Form of Rollup Equations

The three-dimensional shape of the vortex sheet as it rolls up behind a lifting wing is often approximated by considering the sheet at its intersection with the Trefftz plane, which is a plane behind the wing perpendicular to the freestream (see Fig. 1). The Trefftz plane approximation makes it possible to treat the motion of the vortex sheet as a two-dimensional, time-dependent calculation without axial flow. The Betz method does not treat the transition or intermediate stages between the initial vortex sheet behind the wing and the final rolled-up vortex structure. It simply uses the three conservation relations for two-dimensional vortex systems to relate the span-load distribution to the fully-developed vortex structure. In order to achieve a unique result, Betz assumed that the rollup process is independent of any other vortices that may form in the wake and that it is orderly so that the vorticity shed at the wingtip goes into the center of the vortex located at the spanwise centroid of vorticity. Each inboard portion of the sheet then forms a layer of vorticity around all of the previous wrappings until the entire sheet is rolled around the original center, as indicated in Fig. 1.

The spanwise variation of lift on the wing, $l(y)$, is taken to be

$$l(y) = \rho U_\infty \Gamma_w(y) \tag{1}$$

where ρ is the air density, U_∞ the freestream velocity, and $\Gamma_w(y)$ the span-wise variation of circulation or bound vorticity on the wing. The three conservation laws that re-

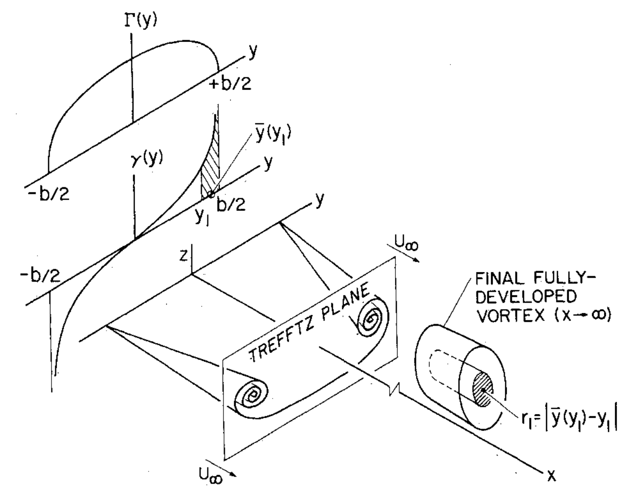

Fig. 1. Schematic diagram of relationship between span loading, $\Gamma_w(y)$, vortex sheet, $\gamma_w(y)$, Trefftz plane, and final rolled-up vortex for one side.

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